Coherent vortex structures in electron temperature gradient driven turbulence

Motoki Nakata¹, Tomo-Hiko Watanabe¹,² and Hideo Sugama¹,²

¹Graduate University for Advanced Studies
²National Institute for Fusion Science

Wendell Horton
The University of Texas at Austin, Institute for Fusion Studies

JIFT workshop 2009, December 14-16th
Outline

- Background and motivations
- Gyrokinetic models and simulation conditions
- Slab-ETG turbulence simulation results
  - Formation of coherent vortex structures and transport reduction
  - Dependence of coherent-structure formation on the parallel flows
- Investigation of toroidal-ETG turbulence
- Summary
**Background and motivations**

- Anomalous heat transport for ions and electrons
  - Ion and electron heat transport observed in magnetically confined plasmas are much larger than the predictions by the collisional transport theory.

- Turbulent transport driven by micro-instabilities
  - Ion temperature gradient (ITG) driven modes
  - Trapped electron modes (TEM)
  - Electron temperature gradient (ETG) driven modes

- Strong anomaly of electron heat transport
  - The gyroBohm scaling predicts smaller electron heat transport: $\chi_e \sim (m_e/m_i)^{0.5} \chi_i$
  - However, experimental observations suggest electron heat transport which is comparable to ion one.

  --> ETG-driven turbulent transport?

---

DIII-D experiment
**Background and motivations**

- Open problems on toroidal ETG turbulence
  - The role of meso-scale structures (e.g. zonal flows, streamers) in nonlinear saturation processes in various parameters such as the magnetic shear $\hat{s}$.  
    - J. Candy et al., PPCF(2007)  
  - Accurate evaluation of the saturation levels

- Investigation of “Slab-ETG turbulence”
  - Primitive system, but closely related to toroidal ETG turbulence with small magnetic shear.
    - High phase-space resolution, parameter-scans
    - Comparison with theoretical models

- Objectives in this study
  - Elucidating what kind of vortex structures enhance or suppress the ETG-turbulent transport? and How?
  - Understanding “phase-space” structures of the ETG turbulence
Gyrokinetic model for slab plasmas

- Gyrokinetic equation for electrons

\[
\left( \frac{\partial}{\partial t} + i k_{||} v_{||} \right) \delta f_{ek}(k_x, k_y, v_{||}) - \frac{c}{B} \sum_{k=k'+k''} b \cdot (k' \times k'') \delta \psi_k \delta f_{ek''}
\]

\[
= -i \left\{ \omega_e \left[ 1 + \eta_e \left( \frac{m_e v_{||}^2}{2T_e} - \frac{1}{2} - \frac{b_k}{2} \right) \right] - k_{||} v_{||} \right\} \frac{e \delta \psi_k}{T_e} + C \delta f_{ek}
\]

- Quasi-neutrality condition with adiabatic ion response

\[
\int dv J_0(k_\perp \rho_e) \delta f^{(g)}_{ek} + n_0 \frac{e \delta \phi_k}{T_e} [1 - \Gamma_0(b_{ek})] = \int dv J_0(k_\perp \rho_i) \delta f^{(g)}_{ik} - n_0 \frac{e \delta \phi_k}{T_i} [1 - \Gamma_0(b_{ik})]
\]

\[
\simeq - n_0 \frac{e \delta \phi_k}{T_i} \ (k_\perp \rho_i \gg 1)
\]

- Entropy balance equation

\[
\frac{d}{dt} (\delta S + W) = Q + D
\]

\( \delta S \): entropy variable \( Q \): turbulent transport

\( W \): potential energy \( D \): collisional dissipation
Simulation conditions

- Shearless slab configuration
  - 3D phase-space \((k_x, k_y, v_{||})\)
    - \(k_z=0\) : homogeneous in \(z\)-direction
    - Averaged over the \(v_{\perp}\) coordinate

- Physical parameters
  \[
  \Theta = \frac{k_{||}L_T}{k_y\rho_{et}} \simeq \frac{L_T}{\rho_{et}}\theta, \quad \eta_e = \frac{L_n}{L_T}
  \]

Linear ETG modes: \(\omega_L, \gamma_L\)

\(\gamma_L\) in Case 1
\(\omega_L/5\) in Case 1
\(\gamma_L\) in Case 2
\(\omega_L/5\) in Case 2

\(\gamma_L^{(\text{case1})} < \gamma_L^{(\text{case2})}\)

Case 1: \(\{\eta_e = 6, \quad \Theta = 1/6\}\)
Case 2: \(\{\eta_e = 10, \quad \Theta = 1/20\}\)
Slab-ETG turbulence simulation results
- A spontaneous transition of the turbulent transport $\chi_e$ is found in Case 2 (more unstable case) while the steady transport is observed in Case 1.

- The onset of the transition to the coherent state is correlated to build up of the strong zonal flow.
Formation of coherent vortex streets

- Contours of potential and temperature fluctuations

- In the turbulent state \((t=2400)\), small-scale fluctuations are developed.
- Finer-fluctuations of \(\delta T\) reflect the development of the fine-scale structures of \(\delta f_{ek}\) in the velocity-space.
- In the coherent state \((t=7800)\), coherent aligned vortices (vortex streets), which are in-phase, are formed both in \(\delta \phi\) and \(\delta T\).

Phase matching in ETG by GF simulations:
T. Matsumoto et al, JPP(2006)
Spectral analysis of fluctuations

- Comparisons of time-averaged spectra in turbulent and coherent states

- The low-wavenumber modes of $|\delta \phi_{k_y}|$ and $|\delta T_{k_y}|$ keep its amplitude while the high-wavenumber modes decrease significantly in the coherent state.

- The decrease of the low-$k$ modes of $|\chi_{e k_y}|$ indicate that the transport reduction is not attributed to the reduction of amplitude of $|\delta \phi_{k_y}|$ and $|\delta T_{k_y}|$. 
Traveling wave solution of HM-$\eta_e$ equation

- Condition for a traveling wave solution of Hasegawa-Mima equation (in the long wavelength limit)

\[
\frac{\partial}{\partial t} \left\{ 1 - \left( 1 + \frac{T_i}{T_e} \right) \nabla^2 \right\} \delta \psi - \frac{\partial}{\partial y} \left( 1 + \frac{\eta_e}{2} \nabla^2 \right) \delta \psi - \left[ \delta \psi , \left( 1 + \frac{T_i}{T_e} \right) \nabla^2 \delta \psi \right] = 0
\]

\[
\tilde{\delta} \psi = \tilde{\delta} \psi(x, y-ut), \quad \frac{\partial}{\partial t} = -u \frac{\partial}{\partial y} = [-ux, \quad \tau : \equiv \frac{T_i}{T_e}]
\]

Condition for a traveling wave solution (TWS-condition)

\[
\Rightarrow \quad \left[ \nabla^2 \tilde{\delta} \psi - \left( \frac{1 + u}{1 + \tau} \right) x , \, \tilde{\delta} \psi - \left\{ u - \frac{\eta_e}{2(1 + \tau)} \right\} x \right] = 0
\]

\[
\equiv S_1 \quad \equiv S_2
\]

- If there is a functional relation between $S_1$ and $S_2$, $\tilde{\delta} \psi$ is a traveling wave solution of HM-$\eta_e$ equation.
- Coherent vortex streets found in the slab ETG turbulence is identified as a traveling wave solution of HM-\(\eta_e\) equation.

- The negative real frequency agrees with the propagation-direction of coherent vortices.
Dependence of the coherent-structure formation on parallel flows
The maximum growth rate distribution on $\Theta$ and $\eta_e$.

Many nonlinear simulations carried out for elucidating how the parallel flow ($$ \Theta = \frac{k_{||}}{k_y} $$) affects the formation of coherent vortex structures.
Summary of the dependence of $W_{ZF}/W_{total}$ on $\Theta = k_{||}/k_y$ for 33 Runs.

- For all $\eta_e$, the large ZF-energy partitions, which is related to transport reduction, are found in “small” and “large” $\Theta$ regions.
Classification of vortex structures

- Potential and temperature fluctuations at $t = 3480$ ($\eta_e = 6$)

- Large $W_{ZF}/W_{total}$ correlates to the formation of coherent structures.
Transport levels vs. $\Theta$

- Dependence of time-averaged $\chi_e$ in turbulent and coherent states on $\Theta$.

- The smaller values of $\Theta$ (small- $k_{||}$) lead to the transport reduction through the formation of coherent vortex structures with strong zonal flows.

$\chi_{\text{trb}}$: time-averaged for $t\sim 700-1500$ (turbulent)

$\chi_{\text{coh}}$: time-averaged for $t\sim 4200-5000$ (coherent)
Investigation of toroidal-ETG turbulence
(preliminary)
**Toroidal ETG turbulence model**

- Gyrokinetic equation for electrons in a non-uniform magnetic field

\[
\frac{\partial}{\partial t} + \left( v_{\parallel} b + v_{de} + \frac{c}{B_0} b \times \nabla \delta \psi \right) \cdot \nabla - \mu b \cdot \nabla \Omega_e \frac{\partial}{\partial v_{\parallel}} - C \right] \delta f_e^{(g)} = - \left( v_{*e} - v_{de} - v_{\parallel} b \right) \cdot \frac{e \nabla \delta \psi}{T_e} F_{Me}
\]

- **GKV code**  T.-H.Watanabe et al. NF(2006)

  - Five-dimensional Eulerian (Vlasov) gyrokinetic simulation
  - Local turbulence simulation in the fluxtube. (Field aligned coordinates)
  - Spectral method for nonlinear $ExB$ advection term
  - Tokamak and Helical geometries

- **Fluxtube in real 3D-space**
Linear simulations

- Linear calculations for toroidal ETG modes

- Toroidal ETG modes with $\hat{s} = 0.1$ show the wider range of unstable modes than that with $\hat{s} = 0.78$ (Cyclone-base case).
- Nonlinear simulation with high phase-space-resolution for $\hat{s} = 0.1$. 

Eigenfunctions for $s=0.1$ and $s=0.78$
Nonlinear simulations

- Entropy balance relation and mode-evolutions \( (\hat{s} = 0.1) \)

![Entropy balance relation (s=0.1)](image1.png)

![Mode-evolutions (s=0.1)](image2.png)

- \( (N_{kx}, N_{ky}, N_z, N_{vpara}, N_{mu}) = (129, 193, 128, 256, 64) \sim 52 \text{ billions grids} \)

- Zonal flow generation is not so strong while the radially elongated streamer mode grow up for \( t > 100 \).
Vortex structures in toroidal ETGs

- Energy partition and contours of potential and temperature fluctuations

- Different elongated structures appear again after the linear mode structures are saturated.
Summary

- Gyrokinetic Vlasov simulation of slab ETG turbulence with high phase-space resolution has been carried out. Then, following results are found:
  1. Formation of coherent vortex streets are observed, then the resultant transport reduction is correlated to the reduction of the phase difference rather than the amplitude.
  2. A traveling wave solution of HM type equation including temperature gradient gives us good approximation to coherent vortex streets.
  3. The vortex structures, i.e., coherent vortex streets, and turbulent vortices and zonal flows with large parallel flows, are classified with the value of $\Theta$ (or $k_\parallel$).
  4. The smaller values of $\Theta$ lead to the transition of $\chi_e$ through the formation of coherent vortex streets with strong zonal flows.

- The detailed analyses of toroidal ETG turbulence and its comparison with the slab ETG case are currently in progress and will be reported elsewhere...