Discontinuous Galerkin modeling of storm surges and scrape-off layer transport

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Abstract

The equations governing the propagation of plasma filaments (blobs) into the scrape-off layer (SOL) share several features with the shallow water equations, which describes the storm surges that occur when hurricanes make landfall. In the SOL, the distance along the field line to the divertor target plate plays a role analogous to the topography in the storm surge problem. The discontinuous Galerkin (DG) method has shown itself to be a robust and flexible algorithm in storm surge modeling. In view of the similarities between the two problems, we are exploring its use for modeling SOL transport. In order to achieve robust numerical solutions, various stability measures are implemented, including: slope-limiting, spectral filtering, artificial diffusion, and regularization screening. The implicit step is preconditioned on an algebraic multigrid, and solved using a generalized minimal residual method (GMRES). We will compare the results of our DG code for simulations of filament propagation with the BOUT++ code, and discuss the advantages of the DG method for modeling the SOL in the presence of magnetic chaos created by resonant magnetic perturbations.

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Outline

• Introduction
• Storm surge modeling
• Chaotic scrape-off layer transport
• Summary
Resonant magnetic perturbations strongly modify the SOL

- Resonant Magnetic Perturbations (RMP) have been shown to mitigate or suppress ELM.
- ITER plans include a set of in-vessel coils to excite RMP.
RMP create magnetic chaos at the edge-SOL boundary

- RMP research is primarily focused on the pedestal; comparatively little is known of the effects of ELM on the SOL, which provides the BC.

M. W. Shafer et al., Nucl. Fusion (2012)
Transport in the SOL is characterized by intermittent filamentary “blobs”

- We will model the evolution of the density $n$ and vorticity $\Omega$ in a flux tube by

\[
\frac{dn}{dt} + (e_z \times \nabla \phi) \cdot \nabla n = -\alpha n + D\nabla^2 n; \\
\frac{\partial \Omega}{\partial t} + (e_z \times \nabla \phi) \cdot \nabla \Omega = \alpha \phi + \beta \frac{\partial n}{\partial y} + \mu \nabla^2 \Omega;
\]

$\Omega = \nabla^2 \phi$

- Here, $\phi$ is the electrostatic potential
- $\beta$ parametrizes the curvature force.
The magnetic geometry enters the SOL model through the sheath-loss rate $\alpha$.

- The parallel end-losses through the sheath are described by the parameter
  
  $$\alpha = \frac{2 \rho_s}{L}$$

  where $L$ is the distance along the field line to the divertor.
Storm surges
Storm surges described by shallow water equations

\[ \frac{\partial H}{\partial t} + \nabla \cdot (Hu, Hv) = 0 \]

\[ \frac{\partial Hu}{\partial t} + \nabla \cdot (Hu^2 + \frac{1}{2}(H^2 - b^2), Hv) = g\zeta \frac{\partial b}{\partial x} + F_x \]

\[ \frac{\partial Hv}{\partial t} + \nabla \cdot (Huv, Hv^2 + \frac{1}{2}(H^2 - b^2)) = g\zeta \frac{\partial b}{\partial y} + F_y \]

Variables:

- Water elevation \( \zeta = H - b \)
- Bathymetry \( b \)
- Water depth \( H \)
- Horizontal velocity \( (u, v) \)
- \( F = (F_x, F_y) \) includes bottom friction, wind stress, atmospheric pressure gradient, tidal potentials, ...

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Bathymetry has multi-sale structure not unlike that of Poincaré maps
The discontinuous Galerkin approach

- The motivation for the discontinuous Galerkin (DG) method is to maintain locality while achieving high-order accuracy on unstructured meshes
  - Unstructured for geometric flexibility
  - High order for accuracy
  - Local for parallel efficiency

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DG simulations are in good agreement with observations
Discontinuous Galerkin methods are attracting growing interest in fusion

SOL transport

Dynamics of plasma filaments (blobs) and role of connection length
There are two approaches to calculating the electrostatic potential

- The potential must be obtained from the time-advanced vorticity by inverting the Laplacian:
  \[ U = \nabla^2 \varphi \]

- The electrostatic potential \( \varphi \) can be represented in a continuous (CG, Gekyl, Hakim) or discontinuous (DG, ArcOn, Michoski) fashion. In the latter case, the Poisson equation is inverted using interior penalty methods.

- The CG representation has advantages for enforcing conservation laws
The ArcOn code handles the blob filaments efficiently.
We have benchmarked the codes with the results of Garcia

- BOUT++ results are in very good agreement.
- The DG results differ from the FD results and from each other by about 20%.
The amplitude of the blob decays rapidly

- BOUT++ is again in good agreement with Garcia’s results
- In the rest of the talk we present results from the FD code BOUT++
- BOUT++ is an object-oriented framework that enables rapid implementation of reduced physics models
We are comparing blob transport through chaotic and smoothed maps.
Density decays exponentially beyond shoulder

- The blob filaments respond primarily to the average of the damping, \(<\alpha>\).
- The chaotic region sets the width of the shoulder between the pedestal and the far-SOL.
- The density in the far-SOL decreases exponentially with decrement \(\lambda_n\).
The width of the shoulder scales with the width of the chaotic layer

The e-folding width $\lambda_n$, by contrast, exhibits no clear dependence on the width of the stochastic layer.
The PDF for the density fluctuations indicates intermittency

- We use the Probability Distribution Function (PDF) for density fluctuations to characterize the turbulence for different $\alpha(x,y)$ functions
- The step-function (good flux surfaces) yields the least intermittent turbulence
- The chaotic $\alpha$ seeds blobs
- Holes are much less likely for the step than the other distributions
Doubling the particle flux increases the intermittency

- The pdf becomes broader as a function of the normalized fluctuation amplitude
- All cases exhibit greater rates of hole creation, but the chaotic field is significantly more effective at generating holes.
- The good flux surfaces are again least effective at generating holes
Conclusion

• The evolution of plasma filaments, or blobs, in a SOL with chaotic field lines bears some similarities to the problem of the calculation of storm surges caused by hurricanes, especially near river deltas.
• The discontinuous Galerkin method is a promising way to solve convection-dominated problems in complex geometries such as the edge and SOL of a tokamak or stellarator.
• We have used the BOUT++ code to guide the development of the DG code ArcOn: we expect its role to become more important in the transition to 3D.
• A chaotic boundary leads to greater intermittency.
• Increasing the flux favors hole creation over blobs.